Effect of weak disorder induced on the first order transition in a p-wave superconductor.

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We investigate the influence of weak disorder on the fluctuation driven first order transition found in a p-wave superconductors. The flow equations for the disorder averaged effective theory is derived in an $4 - \epsilon$ expansion. The fixed point structure of these flow equations and their stability is discussed.

The role of quenched disorder near second order phase transitions have been the subject of intense investigations in the last few decades. These investigations led to the development of the so-called Harris criterion [5] which helps to gauge the relevancy of weak disorder near the clean critical point. Also, there is a significant body of work that deals with strong disorder effects near various critical points. These are generically termed as Griffiths phenomenon [13], which leads to very interesting behavior of various observable quantities like the susceptibility in the vicinity of critical points. However, the role played by quenched impurities near first order transitions has not been so thoroughly investigated. A notable exception to the above statement is the pioneering work of Aizenmann and Wehr, [7] where it was proved that in dimensions, $d \leq 2$ the addition of quenched disorder leads to rounding of the transition. Hui and Berker, [8], also came up with exactly the same conclusion using heuristic Renormalization Group (RG) arguments.

The case in higher dimensions is not so clear. In other words, in certain scenarios, the presence of quenched disorder rounds the first order transition and in other cases it seems that there is no impact on the order of the transition. For instance, the first order transition inherent in a magnetic system with cubic anisotropy can be shown to be unaffected by the addition of weak-disorder [10].

An example in higher dimension wherein disorder does alter the nature of the transition is given by the case of type-II superconductors [4]: In the case of a clean type-II superconductor Halperin and co-workers had shown that the transition was of the fluctuation driven first order type. This conclusion was based on a $4 - \epsilon$ expansion, wherein the RG flows showed runaway behavior. These results for the clean superconductor were also in accordance with the results obtained by Coleman and Wienberg [1] for the same model. In Ref. [2], Boyanovsky and Cardy analyzed the effect of impurities on the fluctuation driven first order transition in such a s-wave superconductor. In particular it was shown in Ref. [2], that if one adds quenched disorder as a random mass term, then as long as the "bare disorder" strength is above a threshold value the first order transition inherent in scalar electrodynamics, (type-II superconductor), gets converted into a second order transition. In other words, if one starts

with a bare disorder value that is below the threshold value then the first order transition persists. These results were obtained in an $\epsilon = 4 - d$ expansion.

In similar vein, using the $\epsilon = 4 - d$ expansion, Blagoeva, et al., [18], studied the nature of the transition in an unconventional superconductor with orthorhombic and cubic anisotropies: They found that the phase transition in these superconductors was predominantly weakly first order. However, in a situation where the orthorhombic anisotropy is absent, they found a new critical fixed point. In [17], the influence of weak randomness on the fluctuation driven first order transition in such unconventional superconductors were studied. They concluded that in an $\epsilon = 4 - d$ expansion the transition in such anisotropic unconventional superconductors remained weakly first order.

Thus, from the discussions of the preceding paragraphs it can be seen that in $d > 2$, the influence of quenched disorder on first order transitions is quite model dependent.

In this brief communication, we look at the effect of quenched non-magnetic impurities on the critical behavior of a p-wave superconductor: The clean (impurity-free), version of this model was recently studied by Qi Li and collaborators [3]. In an $\epsilon = 4 - d$ expansion, when the number of order parameter components, m, are small, they showed that the phase transition in such an unconventional superconductor is also of the fluctuation driven first order variety. These results were analogous to the results obtained for the case of the s-wave superconductor [1], [4], that we had elucidated earlier. To incorporate the effect of disorder, we start with a random mass generalization of the effective action derived by Qi Li and collaborators for the p-wave superconductor. Under this generalization, we set the distance to criticality $t \rightarrow t - \delta(t(\boldsymbol{x}))$, and we obtain:

$$
\begin{aligned}
S = \int d\boldsymbol{x} \Big[& (t - \delta(t(\boldsymbol{x}))) |\boldsymbol{\psi}|^2 + c |\boldsymbol{D}\boldsymbol{\psi}|^2 + u |\boldsymbol{\psi}|^4 + \\
& v |\boldsymbol{\psi} \times \boldsymbol{\psi}^*|^2 + \frac{1}{8\pi\mu} (\boldsymbol{\nabla} \times \boldsymbol{A})^2 \Big].
\end{aligned} \tag{1}
$$

Here, $\boldsymbol{D} = \boldsymbol{\nabla} - i2e\boldsymbol{A}$, is the minimal coupling term. $\boldsymbol{A}$ is the vector potential, and $\boldsymbol{\psi}$, is the m component complex order parameter field. We integrate out the quenched

disorder by using the replica trick. The replica trick essentially results in re-writing the $F = -\{\log Z\}_{\text{dis}} = \lim_{n \to 0} \frac{\{Z^n\}_{\text{dis}} - 1}{n}$. Here, the subscript just implies that one has to perform a disorder average. Now, for calculational ease we assume that the disorder distribution is delta correlated. In other words, we assume that the $\{\delta t(x)\}_{\text{dis}} = 0$, and $\{\delta t(x)\delta t(y)\}_{\text{dis}} = \Delta\delta(x-y)$. We now perform the disorder average and then perform a 1-loop RG calculation. The result of which this analysis gives [12]

$$
\begin{aligned}
\frac{du}{dl} &= \epsilon u + 6e^2 u - 2(m+4)u^2 + 8uv - 8v^2 + 12u\Delta - 3e^4 \\
\frac{dv}{dl} &= \epsilon v + 6e^2 v + 2mv^2 - 12uv + 12v\Delta \\
\frac{d\Delta}{dl} &= \epsilon\Delta + 6e^2\Delta + 8\Delta^2 - 4(m+1)u\Delta + 8v\Delta \\
\frac{de^2}{dl} &= \epsilon e^2 - \frac{m}{3}e^4
\end{aligned}
\tag{2}
$$

The flow equations, Eq. 2, appropriately rescaled, maps onto the case of the disordered s-wave superconductor studied in Ref. [2] when $v = 0$. In the same spirit, it is identical to the flow equations derived in Ref. [3], for the p-wave superconductor when $\Delta = 0$. In what follows, we analyze the fixed points of the above set of equations. We will linearize the flows around the fixed points and thus discuss their stability. Throughout the course of this paper we have suppressed the flow of the distance to criticality t. This is on account of the fact that the t is always a relevant operator. Thus, we look at only the flow in the (u, v, Δ) space. Furthermore, since we are interested in the superconducting transition, we will look for a fixed point with finite value of e^2.
Case 1 : $u^* = v^* = \Delta^* = 0$. This is the Gaussian fixed point which has eigenvalues $\lambda_u = \lambda_v = \lambda_\Delta = 4 - d$. This implies that the Gaussian fixed point is unstable for all $d < 4$, consistent with the naive power-counting.

Next we look at a class of fixed points where one of the coupling constants is constrained to a non-zero fixed point value. Now, the three such fixed points are :

Case 2 : $u^* \neq 0$, $v^* = 0$, and $\Delta^* = 0$.

$$
\begin{aligned}
u^* &= \frac{\epsilon(m+18)}{4m(m+4)}\left[1 \pm \left(1 - \frac{216(m+4)}{(m+18)^2}\right)^{1/2}\right] \\
v^* &= 0 \\
\Delta^* &= 0
\end{aligned}
$$

Consistent with Refs. [3], [4], we call this pair of fixed points the pure s-wave fixed points. It can be shown that this fixed point is physical, (i.e., u^* is real), only when $m > m_c \approx 183$. It can be shown that for physically relevant parameter values, i.e., $m > m_c$, this pair of fixed points are unstable. The most obvious route

to show the instability of the pure s-wave fixed point is to calculate the eigenvalue λ_v, which can be shown to be: $\lambda_v = \frac{1}{u^*}(2(m-2)u^{*2} + \frac{27\epsilon^2}{m^2})$. As can be seen this eigenvalue is always positive in the physically relevant parameter regime.
Case 3: $u^* = 0, v^* \neq 0$ and $\Delta^* = 0$

$$
\begin{aligned}
u^* &= 0 \\
v^* &= -\frac{\epsilon}{2m}\left[1 + \frac{18}{m}\right] \\
\Delta^* &= 0
\end{aligned}
$$

Once, again this is an unstable fixed point. It can be shown that one of the eigenvalues, namely $\lambda_\Delta = -2(m-4)v^*$, is positive for all $m > 4$. Now, for $m < 4$, even though λ_Δ switches signs, it can be shown by following Ref. [3], that this fixed point is unstable.
Case 4: $u^* = 0, v^* = 0$ and $\Delta^* \neq 0$

$$
\begin{aligned}
u^* &= 0 \\
v^* &= 0 \\
\Delta^* &= -\frac{\epsilon}{8}\left[1 + \frac{18}{m}\right]
\end{aligned}
$$

This is an un-physical fixed point, as Δ^*, which is related to the width of the disorder distribution, is negative at this fixed point. Since, this is an un-physical fixed point, we will refrain from discussing its stability here.

Now, we turn our attention to the case where two of the three coupling constants attains non-zero fixed point values:
Case 5: $u^* \neq 0, v^* = 0$ and $\Delta^* \neq 0$

$$
\begin{aligned}
u^* &= \frac{\epsilon}{8(2m-1)}\left[\left[1 + \frac{18}{m}\right] + \sqrt{\left[1 + \frac{18}{m}\right]^2 + \frac{864(2m-1)}{m^2}}\right] \\
v^* &= 0 \\
\Delta^* &= \frac{(m+1)}{2}u^* - \frac{\epsilon}{8}\left[1 + \frac{18}{m}\right]
\end{aligned}
$$

Since the fixed point value $v^* = 0$, we will call this case the disordered s-wave fixed point. It is seen from following Ref. [2], that the λ_u and λ_Δ are both negative for small number of order parameter components. Thus, we need to study the eigenvalue λ_v that we obtain from linearizing around the $v^* = 0$ fixed point. It can be shown that $\lambda_v = (4(m-2)u^* + 4\Delta^*)$. It is obvious that only for the case $m = 1$, will this eigenvalue be negative. Thus, implying that this fixed point is a critical fixed point for $m = 1$. For all other values of m, this fixed point is unstable and the flow is away from this fixed point. It is of little surprise that at the disordered s-wave fixed point, the operator v is irrelevant, as it can be shown that the v term in the action is identically zero. Thus, for $m = 1$, we flow into a critical fixed point first obtained in Ref. [2]. As can be seen from Fig. 1, there is a minimum critical value of the bare disorder strength beyond which
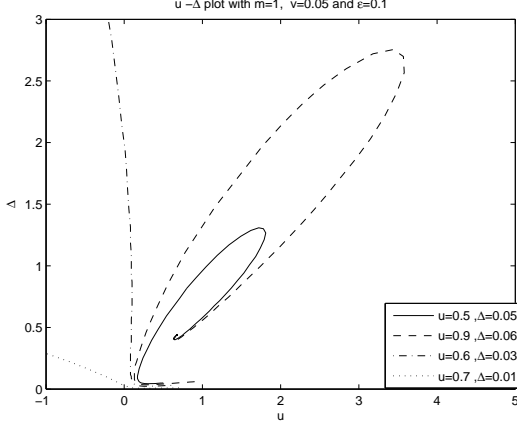
4
4

Case 8: $u^* \neq 0, v^* \neq 0$ and $\Delta^* \neq 0$

$$u^* = \frac{\epsilon}{8m} \left[\frac{(m+18)(m^2 - 34m + 48) \pm (m-6)M_2}{D_2} \right]$$

$$v^* = \frac{\epsilon}{4(m-6)} \left[1 + \frac{18}{m} \right] - \frac{3(m-1)}{(m-6)} u^*$$

$$\Delta^* = \left[\frac{(m+1)}{2} + \frac{3(m-1)}{(m-6)} \right] u^* - \frac{\epsilon(m-4)}{8(m-6)} \left[1 + \frac{18}{m} \right]$$

where $M_2 = (m^4 + 1740m^3 - 48012m^2 + 92448m - 15552)^{1/2}$ and $D_2 = (2m^3 - 55m^2 + 114m - 36)$

These pair of fixed points will be called the disordered p-wave fixed points. As can be seen the expressions for the fixed point values are extremely cumbersome. So is also the case for the eigenvalues that we obtain as a result of linearizing around this fixed point. Thus, for the sake of simplicity, we will discuss the behavior of the system near these fixed points for particular representative values for the number of order parameter components. For small values of m, we can show that the fixed point is unphysical. For example, for $m = 3$, it can be shown that the fixed point value is $u^* = 0.291667\epsilon \pm 0.325071\epsilon\, i, v^* = \pm 0.650142\epsilon\, i$ and $\Delta^* = -0.291667\epsilon$, clearly indicating the unphysical nature of the fixed point. The fixed points are real only when $m \geq 25.1$. Let us look at the behavior of the flow near the disordered p-wave fixed point when we are in this regime. For instance, when $m = 50$, out of the pair of possible fixed points there is only one that is physically relevant. This fixed point is given by $u^* = 0.0109089\epsilon, v^* = -0.0287184\epsilon, \Delta^* = 0.136896\epsilon$. The eigenvalues obtained by linearizing around this fixed point for the case $m = 50$ are given by $-2.49734\epsilon, 0.798418\epsilon \pm 1.57903\epsilon\, i$ and $-\epsilon$, clearly indicating the unstable nature of the fixed point. This state of affairs wherein one gets physically relevant values for the fixed points that are however unstable continues until we reach $m \approx 211$. For m values that are greater than ≈ 211, we can show that the fixed point value of Δ is negative, thus implying that disordered p-wave fixed point goes once again unphysical.

In conclusion, in this brief report we have studied the effect of weak impurities on the first order transition found in a p-wave superconductor. It was seen that for the physically relevant situation of small number of order-parameter components (with $m \neq 1$), the phase transition is still of the fluctuation driven first order type. However, it was also shown that the case $m = 1$, maps itself onto the disordered s-wave superconductor that was studied in Ref. [2].

The conclusion that the critical behavior in a random-mass disordered p-wave superconductor remains weakly first order is encapsulated in Fig. 2, which depicts the numerical solutions of the flow equations, Eq. 2. These are plotted in Fig. 2, wherein the case $m = 1$, is contrasted with the case $m = 2$. For the case $m = 2$, we see the runaway flow persists, indicating that the transition remains first order. Fig. 2, clearly shows that for $m = 1$, we flow into the disordered s-wave fixed point of Ref. [2]. When the number of order parameter components is large i.e., $m > 211$, the pure fixed point found in [3] goes stable. It should be added that our results were obtained in a $4 - \epsilon$ expansion. However, in the case of the ordinary s-wave superconductors with large values of the Ginzburg parameter, κ, it has been shown that the $4 - \epsilon$ expansion is unable to capture the continuous second order nature of the transition seen by using other techniques, [15], [16]. Hence, one had to lay recourse to analytical techniques other than the $4 - \epsilon$ expansion to capture the continuous transition found in superconductors with large κ. For instance, in [14], the RG is performed in exactly $d = 3$, to capture the continuous phase transition. Efforts are on to perform a similar analysis for the case of the p-wave superconductor [12].

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[1] Sidney Coleman and Erick Weinberg, Phys. Rev. D . **7**, 1888 (1973).
[2] Daniel Boyanovsky and John L Cardy, Phys. Rev. B. **25**, 7058 (1982).
[3] Qi Li, D. Belitz and J. Toner, arXiv:0808.3821
[4] B. I. Halperin, T. C. Lubensky, and S. Ma, Phys. Rev. Lett. **32**, 292, 1974.
[5] A. B. Harris, J. Phys. C **7**, 1671, (1974).
[6] Y. Imry and M. Wortis, Phys. Rev. B **19**, 3581 (1979).
[7] M. Aizenman and J. Wehr, Phys. Rev. Lett. **62**, 2503 (1989).
[8] K. Hui and A. N. Berker, Phys. Rev. Lett. **62**, 2507 (1989).
[9] J. Rudnick, Phys. Rev. B **18**, 1406 (1978).
[10] R. Narayanan and T. Vojta, Phys. Rev. B **63**, 014405 (2001)
[11] Dorogovstev, Phys. Lett. **76 A**, 169 (1980).
[12] Priyanka Mohan and R. Narayanan, unpublished notes.
[13] R.B. Griffiths, Phys. Rev. Lett. **23**, 17 (1969).
[14] I. F. Herbut, and Z. Tesanovic, Phys. Rev. Lett. **76**, 4588 (1996).
[15] C. Dasgupta, and B. I. Halperin, Phys. Rev. Lett. **47**, 1556 (1981).
[16] J. Bartholomew, Phys. Rev. B. **28**, 5378 (1983).
[17] G. Busiello, L. De Cesare, Yonko T. Millev, I. Rabuffo, and Dimo I. Uzunov, Phys. Rev. B **43**, 1150 (1991).
[18] E. J. Blagoeva, G. Busiello, L. De Cesare, Y. T. Millev, I. Rabuffo, and D. I. Uzunov, Phys. Rev. B **42**, 6124 (1990).